American University of Beirut
Statistics 230 Sections 3 and 4
Test1, November 4^{th}, 2010
Type BBB

NAME:

Answer the following question by giving a brief justification. You can use the other blank side of the page to finish your work if needed.

1. There are ten different types of candy in a box and there are three candies of each type. five candies are chosen at random from the box. What is the probability that they include one whole set of candies and two distinct candies? e.g. three of type 1, and two other distinct candies?

   There are 10 ways to choose first type (of which we get all candies). Then we choose two more types.

   The answer is then: $\dfrac{10\binom{3}{3}\binom{9}{2}\binom{3}{1}\binom{3}{1}}{\binom{30}{5}}$.

2. four girls and nine boys are randomly seated linearly on thirteen chairs. What is the probability that no two girls are seated on adjacent seats.

   Notice that only the gender (B or G) is significant. The total number of ways is $\binom{13}{4}$. To count the permissible cases, count the two cases where the last person is a boy or a girl respectively. When the last one is B, couple the four G's as GB, so we have the four 'objects' GB and the remaining 5 Bs. We have $\binom{9}{4}$ ways to shuffle them. When the last person is G, fix one G there and couple the remaining 3 Gs with 3 Bs to get GB, GB GB. There are $\binom{9}{3}$ ways. The answer is therefore

   $$\frac{\binom{9}{4}+\binom{9}{3}}{\binom{13}{4}}.$$

   Alternatively, if you replace the letter G by the symbol | and B by O, we can think of placing the Os between or around the |s so that no two |s are adjacent. So the three inner spots between the —s must be nonempty. Assign 3 Os for these spots we have 6 more Os to be distributed in 5 spots, there are $\binom{10}{4}$ ways to do that.

3. Five unrelated people get into an elevator of a building with 8 floors. What is the probability that exactly two people lend on the same floor while the other three people all lend on different floors?

Hint: first settle the questions: which two people? and which floor gets them? then worry about the remaining people and floors.

$$\frac{\binom{5}{2} \times 8 \times 7 \times 6 \times 5}{8^5}.$$

4. In a repeated experiment of tossing a coin twice, find the probability that we observe two tails before observing two heads. Note that every trial is to toss twice, so if you get heads, tails you repeat the two tosses. Assume that the coin has probability 0.4 of heads.

   Since the tossing stops only when either TT occurs or HH occurs, we need to find $P[TT|TTorHH] = P[TT]/P[TTorHH] = 0.36/(0.36 + 0.16) = 36/52 = 9/13$.

5. A system has n identical components that operate independently of each other. The system fails when all components fail simultaneously. The probability of failure for each component is 0.2, let X be the number of failed components on a given day. find the number of components needed so that the standard deviation of X is 4.

6. A pair of fair dice is rolled. Player A bets on a sum of six while player B bets on a sum of five. If the sum is neither six nor seven the dice are rolled again and again until one of the players wins. Find the expected number of times the dice are rolled.

7. The moment generating function of a random variable is given by $M(t) = c(e^{2t}+2e^t+1)^3$. Find the probability of $X = 1$.

8. Three events are labeled A, B and C. Write an expression to denote the event that at least one of these three events is actually observed.

   It is simply $A \cup B \cup C$.

9. A certain disease is present in 0.01% of a population. A test used to detect this disease is 99.99% sensitive (detects the disease when it is present), and 99% specific (gives a negative result when the disease is not present). A randomly selected individual is tested positive. The probability that this individual has the disease is:

   We want $P[D|T^+]$ where D is the event that the disease is present. we have $P[T^+|D] = 0.9999$, $P[T^-|D^c] = 0.99$ and $P[D] = 0.001$. So $P[T^-|D] = 0.0001$ and $P[T^+|D^c] = 0.01$. Now by Bayes Theorem,

$$P[D|T^+] = \frac{P[D] \times P[T^+|D]}{P[D] \times P[T^+|D] + P[D^c] \times P[T^+|D^c]}.$$

Plugging in the numerical values we get 1/101 or 0.01%.

10. Players A and B respectively have probabilities 0.6 and 0.4 of winning a point when they play against each other. If they keep playing independent games until one player has three points more than the other, what is the probability that they will finish in five games or less?

    Winning in three games, 3 As or 3 Bs: $0.6^3 + 0.4^3$.

    Winning in five games, (4 As and 1 B) or (1 A and 4 Bs): $\binom{4}{1} 0.4^4 \times 0.6 + \binom{4}{1} (0.6)^4 \times 0.4$. Note here that, e.g., for A to win in five games, the fifth has to be A and the first 4 must include one 1 B.

    The answer is the sum of all the following terms.